Rigorous Modal Analysis of the Asymmetric Rectangular Iris in Circular Waveguides

Reinhard Keller and Fritz Arndt, *Fellow, IEEE*

Abstract—**The rigorous field theory analysis is presented for the rectangular iris in circular waveguides as well as for rectangular iris coupled circular waveguide resonators. The theory is based on the full-wave mode-matching method for the key-building block discontinuity circular waveguide to a noncentric smaller rectangular waveguide, associated with the generalized S-matrix technique. Arbitrary iris location and finite thickness are rigorously taken into account. The scattering parameters of a single transition and of a rectangular iris coupled one-resonator filter of about 12 GHz resonance frequency are presented as calculation examples. The theory is verified by comparison with measurements.**

I. Introduction

Rectangular iris coupling elements in circular waveguides have found widespread applications in many advanced microwave circuits, such as in the design of microwave filters and multiplexing networks for modern satellite communication systems [1]. When stringent requirements are placed on the system, when additional fine tuning by screws for the filters should be avoided, or when higher-order mode coupling effects have to be utilized, the need arises for accurate design tools which allow to take into account all relevant parameters, like the finite thickness of the irises and the higher-order mode interaction between them.

For rectangular waveguides, the rigorous analysis of rectangular, circular and cross-shaped irises is already well-known, [2]–[8]. The solution of the junction problem of a circular waveguide to a centered smaller rectangular waveguide has been reported recently [9]. However, for rectangular irises in circular waveguides, the designers are still left to approximate equivalent circuit formulas [10]. Although, in principle, the inverse transition (rectangular to a smaller circular waveguide) can be utilized [7] to solve the problem by introducing an adequately large artifical rectangular waveguide intermediate section of zero length, it is advantageous to calculate the iris element directly.

The purpose of this letter is to present a rigorous field-theory analysis for the arbitrarily located rectangular iris in a circular waveguide which is based on the full-wave mode-matching method for the key-building block discontinuity circular to smaller rectangular waveguide (Fig. 1) directly. In contrast to [9], the general noncentered case is included in order to allow for the investigation of more universal iris locations and for possible fundamental-to-higher order mode coupling effects. Composite structures, such as the rectangular iris in a circular waveguide and rectangular iris coupled circular waveguide filters, are rigorously modeled by the generalized scattering matrix technique. The efficiency of the method is demonstrated by analyzing a rectangular iris coupled one-resonator filter with a resonance frequency of about 12 GHz. The theory is verified by comparison with measurements.

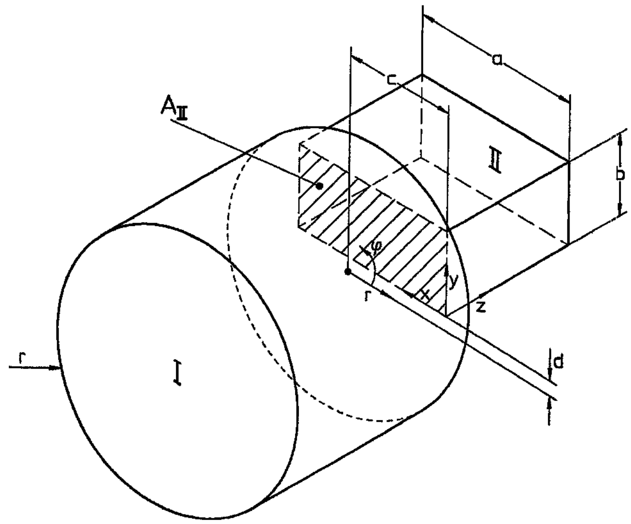

Fig. 1. Key-building block discontinuity circular waveguide to noncentric smaller rectangular waveguide.

II. Theory

The rectangular iris element of finite thickness t is decomposed into the key-building block discontinuity circular to smaller rectangular waveguide (Fig. 1), a homogeneous rectangular waveguide section of length t, and the inverse key-building block structure, associated with the generalized S-matrix technique. Note that for deriving the scattering matrix of the inverse structure merely the port designations of the original structure has to be interchanged.

In the key-building block discontinuity, the fields in the subregions ν = I, II (Fig. 1)

$$\vec{E}^{\nu} = \nabla \times (A^{\nu}_{Hz}\vec{e}_z) + \frac{1}{j\omega\epsilon}\nabla \times \nabla(A^{\nu}_{Ez}\vec{e}_z)$$
$$\vec{H}^{\nu} = \nabla \times (A^{\nu}_{Ez}\vec{e}_z) - \frac{1}{j\omega\mu}\nabla \times \nabla \times (A^{\nu}_{Hz}\vec{e}_z) \quad (1)$$

Manuscript received February 17, 1993.
R. Keller is with ANT Nachrichtentechnik GmbH, D-7150 Backnang, Germany.
F. Arndt is with the Microwave Department, University of Bremen, Kufsteiner Str., NW1, D-2800 Bremen, Germany.
IEEE Log Number 9209407.

1051-8207/93$03.00

are derived from the z-components of two vector potentials

$$A_{Hz}^{\nu} = \sum_{m=1}^{M} (\sqrt{Z}_{Hm}^{\nu}) \cdot N_{Hm}^{\nu} \cdot T_{Hm}^{\nu} \cdot [a_{Hm}^{\nu} \exp(-\gamma_{Hm}^{\nu} z) + b_{Hm}^{\nu} \exp(+\gamma_{Hm}^{\nu} z)]$$
$$A_{Ez}^{\nu} = \sum_{m=1}^{M} (\sqrt{Y_{Em}^{\nu}}) \cdot N_{Em}^{\nu} \cdot T_{Em}^{\nu} \cdot [a_{Em}^{\nu} \exp(-\gamma_{Em}^{\nu} z) - b_{Em}^{\nu} \exp(+\gamma_{Em}^{\nu} z)], \quad (2)$$

with the wave impedances or admittances

$$Z_{Hm}^{\nu} = (j\omega\mu_{\nu})/(\gamma_{Hm}^{\nu}), \quad Y_{Em}^{\nu} = (j\omega\epsilon_0)/(\gamma_{Em}^{\nu}), \quad (3)$$

and the propagation factors $\gamma_{Hm}^{\nu}, \gamma_{Em}^{\nu}$ of the TE− and TM−modes, respectively.

T_{Hm}^{ν} and T_{Em}^{ν} are the cross-section eigenfunctions of the corresponding circular or rectangular waveguide regions I, II. N_{Hm}^{ν}, N_{Em}^{ν} are the normalization factors so that the relation of the still unknown TE− and TM−mode wave amplitudes of the forward and backward waves, $a_{H,E}^{\nu}$, $b_{H,E}^{\nu}$, yields directly the corresponding modal scattering matrix relations by matching the tangential field components of regions I and II at the common interface:

$$\begin{bmatrix} (b^I) \\ (b^{II}) \end{bmatrix} = (S) \begin{bmatrix} (a^I) \\ (a^{II}) \end{bmatrix}. \quad (4)$$

The coupling integrals in (4) are of the form

$$C_m = \iint_{A_{\text{II}}} B_m(k_c^{\text{I}} r) \cdot \begin{bmatrix} \cos(i\varphi) \\ \sin(i\varphi) \end{bmatrix} \cdot \sin(k_x^{\text{II}} x) \cdot \sin(k_y^{\text{II}} y) dA_{\text{II}}, \quad (5)$$

where B_m is the Bessel-function of the first kind, k_c^{I} is the cut-off wavenumber in the circular waveguide, and $k_x^{\text{II}}, k_y^{\text{II}}$ are the wavenumbers in x and y direction in the rectangular waveguide.

About 40 TM− and 40 TM−modes (both cosine- (c−) and sine- (s−) types) in the circular waveguide section have turned out to yield already sufficient asymptotic behavior. In the rectangular waveguide section, the number of modes are chosen according to the geometrical ratio $a/(2r_0)$. The final design data are provided by expansion into $\text{TE}_{mn}^{\text{c,s}}$− and $\text{TM}_{mn}^{\text{c,s}}$−modes up to $m = 9$, $n = 9$.

III. Results

For verification of the theory for the single key-building block, Fig. 2 presents the calculated and measured fundamental mode (TE_{10}) return loss curve of a transition rectangular waveguide (WR-75 housing: 19.05 mm × 9.525 mm) to a larger circular waveguide ($2r_0 = 23.5$ mm), for the centered case (solid line). Very good agreement with measurements may be stated. The influence of the noncentered location is demonstrated by the dashed curve where the iris location is offset by 2.1 mm. For this case, in the circular waveguide, all $\text{TE}_{mn}^{\text{c,s}}$− and $\text{TM}_{mn}^{\text{c,s}}$− modes up to $m = 9$, $n = 9$, and in the rectangular waveguide section all TE_{mn}− and TM_{mn}−modes up to $m = 7$, $n = 4$ are taken into account.

Fig. 3 shows the insertion and return loss plots versus frequency of a rectangular iris coupled one-resonator circular waveguide filter with WR-75 rectangular waveguide input ports. For this example, the main mode in the circular cavity is the TE_{113}−mode. The filter has been fabricated by milling the 1-mm thick irises. The dimensions are presented in the figure legends. Excellent agreement with measurements is observed.

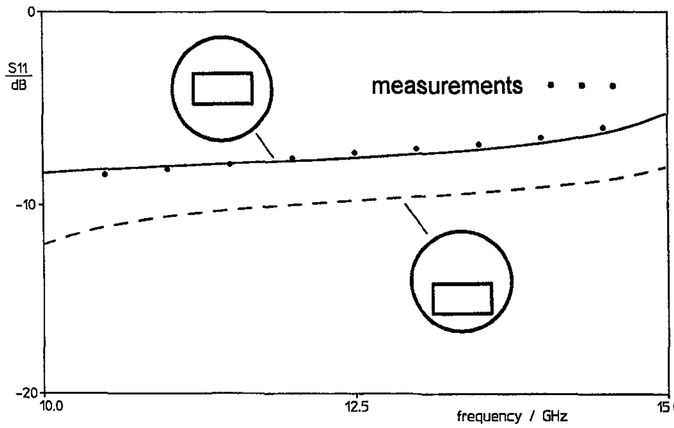

Fig. 2. Return loss vs. frequency of transitions smaller rectangular waveguide (WR 75: 19.05 mm × 9.525 mm) to circular waveguide ($2r_0 = 23.5$ mm), centered (———), offset by 2.1 mm (- - -). Comparison with measurements (· · ·).

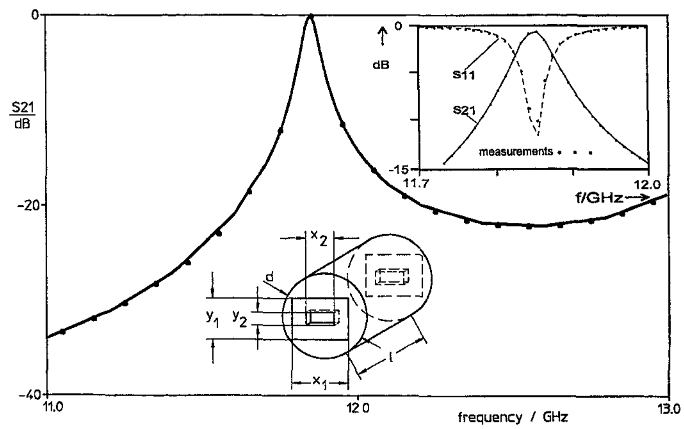

Fig. 3. Insertion and return loss of a rectangular-iris coupled one-resonator circular waveguide filter. The rectangular waveguide ports are WR 75: $x_1 = 19.05$ mm, $y_1 = 9.52$ mm. Resonator dimensions: $d = 26.5$ mm, $l = 45$ mm. Iris dimensions: $x_2 = 9.3$ mm, $y_2 = 3$ mm, thickness $t = 1$ mm. Comparison with measurements (· · ·).

IV. Conclusion

A rigorous and efficient mode-matching method is presented for the accurate design of arbitrarily located rectangular irises in circular waveguides. Since all relevant design parameters, such as finite iris thickness and higher order mode coupling effects are taken into account, very good agreement with measured results is obtained.

References

[1] C. Kudsia, R. Cameron, and W.-C. Tang, "Innovations in microwave filters and multiplexing networks for communications satellite systems," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1133–1149, June 1992.

[2] H. Patzelt and F. Arndt, "Double-plane steps in rectangular waveguides and their applications for transformers, irises, and filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 771–776, May 1982.

[3] H. D. Knetsch, "Wellentypwandler und Filter mit Rechteck- und Rundhohlleiterelementen," *Nachrichtentechnische Zeitschrift*, vol. NTZ-23, pp. 57–63, Feb. 1970.

[4] J. D. Wade and R. H. McPhie, "Scattering at circular-to-rectangular waveguide junctions", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1085–1091, Nov. 1986.

[5] R. W. Scharstein and A. T. Adams, "Galerkin solution for the thin circular iris in a TE_{11}-mode circular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 106–113, Jan. 1988.

[6] G. B. Eastham, and K. Chang, "Analysis and closed-form solutions of circular and rectangular apertures in the transverse plane of a circular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 718–723, Apr. 1991.

[7] F. Arndt and U. Papziner, "Modal-S-matrix design of microwave filters composed of rectangular and circular waveguide elements," in *IEEE MTT-S Int. Symp. Dig.*, 1991, pp. 535–538.

[8] R. Ihmels and F. Arndt, "Rigorous modal S-matrix analysis of the cross-iris in rectangular waveguides," *IEEE Microwave Guided Wave Lett.*, vol. 2, pp. 400–402, Oct. 1992.

[9] R. H. MacPhie *et al.*, "Full wave scattering analysis of the junction of a circular waveguide and a smaller rectangular waveguide," in *URSI Radio Sci. Dig.*, Chicago, IL, July 1992, p. 216.

[10] G. L. Mathaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures.* New York: McGraw-Hill, 1964.